Solutions must be typeset in LaTeX. Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. Any collaboration should involve only discussion; all writing **must** be done individually. You are free to use the result from any problem on this (or previous) assignment as a part of your solution to a different problem even if you have not solved the former problem.

Problem 1. Show that it is possible to color the edges of K_n with $O(\sqrt{n})$ many colors so that there are no monochromatic triangles.

Problem 2. Recall that a k-AP is a set of the form $\{x, x+d, x+2d, \ldots, x+(k-1)d\}$. Fix positive integers $p \geq q$. Prove that there is a constant $C = C(p, q)$ such that it is possible to color $[n]$ with at most $Cn^{1/(p-q+1)}$ many colors so that every p-AP sees at least q different colors.

N.b. If $q \leq \lceil p/2 \rceil$, then it is possible to color $[n]$ with $n^{o(1)}$ many colors so that every p-AP sees at least q different colors. I conjecture that it is necessary to use $n^{\Omega(1)}$ many colors to accomplish the same feat when $q = \lceil p/2 \rceil + 1$.

Problem 3. Prove that there is a constant $c > 0$ with the following property: For any positive integer k and any graph G, if $L\colon V(G) \to \binom{\mathbb{N}}{k}$ is any list-assignment with the property that for any vertex v and any color $t \in L(v)$, the color t appears in the list assigned to at most ck of v's neighbors, then G has a proper coloring where each vertex v is assigned a color from its list $L(v)$.

Problem 4. Let G be a graph with max degree at most Δ. Prove that there is an edge-coloring of G using $O(\Delta)$ many colors such that every cycle sees at least three different colors.