

In this homework, we will prove a fact that we've all known since grade school, namely, a trick to determine whether a number is divisible by 3. Let  $a = "a_n a_{n-1} \cdots a_1 a_0"$  be a number where  $a_i$  is the  $i$ th digit of  $a$  when written in decimal form; in particular,  $a = \sum_{i=0}^n a_i \cdot 10^i$ . For example if  $a = 1045$ , then  $a_0 = 5$ ,  $a_1 = 4$ ,  $a_2 = 0$  and  $a_3 = 1$ .

Justify all answers!

**(6 pts)**

- (1) [+4] Let  $a = "a_n a_{n-1} \cdots a_1 a_0"$  be an integer where  $a_i$  is the  $i$ th digit of the decimal form of  $a$ . Prove that  $3 \mid a$  if and only if  $3 \mid \sum_{i=0}^n a_i$ . (Hint: modular arithmetic may be helpful)
- (2) [+2] Let  $n \in \mathbb{N}$  have  $n \geq 2$ . Prove that  $(n-1)^{-1} \equiv n-1 \pmod{n}$ .