Justify all answers!

(12 pts)

(1) [+4] Let A, B, C, D be sets. Prove that, if $A \cup B \subseteq C \cup D$ and $C \subseteq A$ and $A \cap B = \varnothing$, then $B \subseteq D$.

(2) (a) [+3] Prove that if A, B are sets, then $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

(b) [+1] Must it be the case that $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$?

(3) [+4] Prove that $\bigcup_{n \in \mathbb{N}} [n] = \mathbb{N}$.