This document is from https://mathematicaster.org/teaching/discrete2026/hw/hw1.pdf

Solutions must be typeset in LaTeX. Collaboration is allowed, but must be fully acknowledged. Any collaboration should involve only discussion; all writing **must** be done individually. Use of generative AI is **forbidden**. You are free to use the result from any problem on this (or previous) assignment as a part of your solution to a different problem even if you have not solved the former problem.

**Problem 1.** Tell me a little bit about yourself!

- What's your year and major?
- What other classes are you taking this semester?
- What are you most excited to learn in this class?
- Is there anything specific that you'd like to cover in this class?

**Problem 2.** A function $f\colon [m] \to [n]$ is *weakly increasing* if $i \leq j \implies f(i) \leq f(j)$. How many weakly increasing functions are there from $[m]$ to $[n]$? Justify your answer with a bijective argument.

**Problem 3.** Give a combinatorial proof that

$$\binom{m_1+m_2}{n} = \sum_{k=0}^{n} \binom{m_1}{k}\binom{m_2}{n-k}.$$

**Problem 4.** Let $f_i(n)$ denote the number of permutations $\pi \in S_n$ which have exactly $i$ fixed points (that is, $\pi(x) = x$ for exactly $i$ choices of $x$). Give combinatorial proofs of the following identities:

1. $n! = \displaystyle\sum_{i=0}^{n} f_i(n)$.
2. $n! = \displaystyle\sum_{i=0}^{n} i \cdot f_i(n)$.

**Problem 5.** Use the binomial theorem and some calculus to find formulas for the following expressions:

1. $\displaystyle\sum_{k=0}^{n} \binom{n}{k} k$
2. $\displaystyle\sum_{k=0}^{n} \binom{n}{k} \frac{1}{k+1}$