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Solutions must be typeset in $\text{\LaTeX}$. Collaboration is allowed, but must be fully acknowledged. Any collaboration should involve only discussion; all writing **must** be done individually. Use of generative AI is **forbidden**. You are free to use the result from any problem on this (or previous) assignment as a part of your solution to a different problem even if you have not solved the former problem.

Problem 1. Show that for any fixed $d \geq 0$,

$$\sum_{k=1}^n k^d \sim \frac{n^{d+1}}{d+1} \quad \text{as } n \rightarrow \infty.$$

Problem 2. Let $d(n)$ denote the number of divisors of n (including 1 and n) and define $\bar{d}(n)$ to be the average of $d(i)$ over $i \in [n]$. Prove that $\bar{d}(n) \sim \log n$.

Problem 3. Improve our in-class lower-bound on the harmonic numbers to $H_n \geq \log n + \frac{1}{2}$.

Hint: Trapezoids!