**Problem 1.** How many words of length $n$ are there with $k$ many 0's and $n-k$ many 1's?

How many words of length $n$ are there with $k$ many 0's and $\ell$ many 1's and $n-k-\ell$ many 2's?

Generalize this: How many words of length $n$ are there with $k_i$ many $i$'s for $i \in \{0, \ldots, r\}$ (where $n = k_0 + \cdots + k_r$)?

**Problem 2** (Lattice paths)**.** A lattice path on the $m \times n$ grid is a sequence of steps starting at $(0,0)$, ending at $(m,n)$ which steps either up $((x,y) \mapsto (x,y+1))$ or right $((x,y) \mapsto (x+1,y))$ at every time. Determine, with proof, the number of lattice paths on the $m \times n$ grid.

**Problem 3** (Fibonacci numbers)**.** Recall that the Fibonacci numbers are defined by $f_0 = f_1 = 1$ and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$. Show that the number of ways to tile a $2 \times n$ strip with dominoes (i.e. $2 \times 1$ or $1 \times 2$ tiles) is precisely $f_n$.

**Problem 4** (Even vs odd)**.** Fix $n \geq 1$. Prove that the number of even-sized subsets of $[n]$ is the same as the number of odd-sized subsets of $[n]$.

**Problem 5** (Positive integer combinations)**.** In class, we counted the number of non-negative integer solutions to $x_1 + \cdots + x_k = n$. How many *positive* (i.e. $x_i \geq 1$) integer solutions are there to this equation?

**Problem 6** (Hockey-stick identity)**.** Prove that

$$\binom{n+1}{r+1} = \sum_{k=r}^{n} \binom{k}{r}$$