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Problem 1. Prove that $1 + x \leq e^x$ for all $x \in \mathbb{R}$. This will be an incredibly important inequality once we start to study the probabilistic method.

Hint: Think about the derivative of $f(x) = e^x - x - 1$.

Problem 2. Prove the bounds

$$\left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \left(\frac{en}{k}\right)^k.$$

Problem 3. Show that $\displaystyle\sum_{k=1}^{n} \frac{1}{\sqrt{k}} \sim 2\sqrt{n}$.

Problem 4.

- Fix a non-negative integer k and find an asymptotic for $\binom{n}{k}$ as $n \to \infty$.
- Fix a number $0 < \alpha < 1$ and find an asymptotic for $\binom{n}{\alpha n}$ as $n \to \infty$. Your answer should have the form $\frac{g(\alpha)}{\sqrt{n}} 2^{h(\alpha)n}$ for some functions g, h. The function h is called the *binary entropy function*.