Problem 1. Recall that a graph is a pair $G = (V, E)$ where $E \subseteq \binom{V}{2}$. A vertex in G is said to be *isolated* if no edges are incident to it. Find a formula for the total number of (labeled) graphs with $V = [n]$ having no isolated vertices.

Problem 2. Recall that the Stirling numbers (of the second kind) $\left\{ {n \atop k} \right\}$ count the number of ways to partition an n-element set into k non-empty pieces. Prove that

$$\left\{ {n \atop k} \right\} = \frac{1}{k!}\sum_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n = \frac{1}{k!}\sum_{i=0}^{k}(-1)^{k-i}\binom{k}{i}i^n.$$

Hint: Start by justifying that $k!\left\{ {n \atop k} \right\}$ counts the number of surjections from $[n]$ to $[k]$.

Problem 3 (Alternative proof of the inclusion–exclusion formula)**.**

- Prove that, for any numbers[1] $x_1, \dots, x_n$,

$$\prod_{i=1}^{n}(1 + x_i) = \sum_{I \subseteq [n]} \prod_{i \in I} x_i$$

- Justify that for any sets A, B, we have $\mathbf{1}[\overline{A}] = 1 - \mathbf{1}[A]$ and $\mathbf{1}[A]\mathbf{1}[B] = \mathbf{1}[A \cap B]$.
- Derive the inclusion–exclusion formula by using $x_i = -\mathbf{1}[B_i]$

[1] Really, any elements in a commutative ring containing 1.