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Problem 1. Show that among any $n+1$ integers, there is some a, b with $a-b$ divisible by n.

Problem 2. Fix any positive number d. Show that if $\mathbb{R}^2$ is colored red and blue, then there is some pair of points at distance d of the same color.

Problem 3. K_n is the graph with n vertices and all $\binom{n}{2}$ edges. Show that if the edges of K_6 are colored red and blue, then there is a monochromatic triangle. Show that there is a way to color the edges of K_5 red and blue without creating a monochromatic triangle.

Problem 4. A chess master has 77 days to prepare for a tournament. They play at least one game per day but at most 132 games in total over these 77 days. Prove that there is some sequence of consecutive days over which they play exactly 21 games.