These notes are from https://mathematicaster.org/teaching/graphs2022/extra_03-01.pdf

Here is one very useful fact about edge-connectivity to which the book alludes but never really states.

For a graph $G = (V, E)$ and disjoint subsets $A, B \subseteq V$, we write $E[A, B]$ to denote the set of edges of G with one vertex in A and the other vertex in B. Observe that if $V = A \sqcup B$ and A, B are both non-empty, then $E[A, B]$ is an edge-cut of G. It turns out that this is morally a biconditional statement.

Lemma 1. *Let $G = (V, E)$ be a graph on at least two vertices.*

1. *If $S \subseteq E$ is an edge-cut, then there is a partition $V = A \sqcup B$ with A, B non-empty such that $S \supseteq E[A, B]$.*
2. *If $S \subseteq E$ is a minimum edge-cut, then there is a partition $V = A \sqcup B$ with A, B non-empty such that $S = E[A, B]$.*

Proof. Since S is an edge-cut and $G \ncong K_1$, $G - S$ is disconnected. We can thus partition $V = A \sqcup B$ such that A and B are non-empty and no edges of $G - S$ cross between A and B. Thus, if $e \in E[A, B]$, then we must have $e \in S$; i.e. $E[A, B] \subseteq S$.

Now suppose that S is a minimum edge-cut and take A, B from earlier and suppose for the sake of contradiction that $S \neq E[A, B]$. Since $S \supseteq E[A, B]$, this can happen only if there is some $e \in S \setminus E[A, B]$. Consider $S' = S \setminus \{e\}$. Since $e \notin E[A, B]$, we see that $G - S' = G - S + e$ still has no edges crossing between A and B. Thus $G - S'$ is also disconnected and so S' is an edge-cut of G, contradicting the minimality of the edge-cut S. □

Consequentially, if G has at least two vertices, then

$$\lambda(G) = \min\big\{|E[A, B]| : V(G) = A \sqcup B \text{ and } A, B \neq \varnothing\big\}.$$