

This assignment is meant to mostly be a review of things with which you should be familiar before taking this class. These problems will be graded only for completion, but please still take the time to do each of them correctly. If you are unfamiliar with any of these concepts, please take the time to familiarize yourself; we will go over these again in class, but it will be done quickly.

Throughout, i is the imaginary unit, which satisfies $i^2 = -1$.

(10 pts)

(1) Find the complex conjugate and multiplicative inverse of the following complex numbers. Write your answers in the form $a + bi$.

(a) **[+1]** $4 + 3i$.

Solution: $\overline{4 + 3i} = 4 - 3i$.

$$(4 + 3i)^{-1} = \frac{\overline{4+3i}}{|4+3i|^2} = \frac{4-3i}{4^2+3^2} = \frac{1}{25}(4 - 3i).$$

□

(b) **[+1]** $\frac{1}{2}(-1 + i)(7 - 17i)$.

Solution: $\frac{1}{2}(-1 + i)(7 - 17i) = 5 + 12i$.

$$\overline{5 + 12i} = 5 - 12i$$

$$(5 + 12i)^{-1} = \frac{\overline{5+12i}}{|5+12i|^2} = \frac{1}{169}(5 - 12i).$$

□

(2) Locate the roots of the following polynomials.

(a) **[+1]** $x^2 + 2$.

Solution: $x = \pm\sqrt{2}i$.

□

(b) **[+1]** $ix^2 + 3(1 - i)x - 3 + 2i$.

Solution: $x = 1, 2 + 3i$.

□

(c) **[+1]** $(2 + i)x^3 - (9 + 2i)x^2 + (9 - 3i)x$.

Solution: $x = 0, 3, 1 - i$.

□

(3) Solve the following systems of linear equations in whatever way you please.

(a) **[+1]** $\begin{cases} x + y = 2 \\ x - y = 1 \end{cases}$.

Solution: $x = 3/2, y = 1/2$.

□

(b) **[+1]** $\begin{cases} (1 - i)x + 2iy = 1 \\ (2 + 5i)x - 2iy = 0 \end{cases}$.

Solution: $x = \frac{1}{25}(3 - 4i), y = \frac{1}{50}(7 - 26i)$.

□

(4) For the following matrices A, B , compute $A + B$, AB and BA or explain why these computations cannot be done.

(a) $[+1]$ $A = \begin{bmatrix} 5 & -4 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ -3 & 2 \end{bmatrix}$.

Solution: $A + B = \begin{bmatrix} 5 & -3 \\ -3 & 3 \end{bmatrix}$.

$$AB = \begin{bmatrix} 12 & -3 \\ -3 & 2 \end{bmatrix}.$$

$$BA = \begin{bmatrix} 0 & 1 \\ -15 & 14 \end{bmatrix}.$$

□

(b) $[+1]$ $A = \begin{bmatrix} 5 - 2i & -4 \\ i & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 - i \\ 1 \end{bmatrix}$.

Solution: $A + B$ cannot be computed since A and B are not of the same dimensions.

$$AB = \begin{bmatrix} -1 - 7i \\ 2 + i \end{bmatrix}.$$

BA cannot be computed since the number of columns of B is not the same as the number of rows of A .

□

(c) $[+1]$ $A = \begin{bmatrix} 3 & -3 \\ 2 & -2 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

Solution: $A + B$ cannot be computed since A and B are not of the same dimensions.

$$AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

$$BA = \begin{bmatrix} 6 & -6 \\ 6 & -6 \end{bmatrix}.$$

□